Vortex Induced Wing Loads

L. T. Filotas*
University of Maryland, College Park, Md.

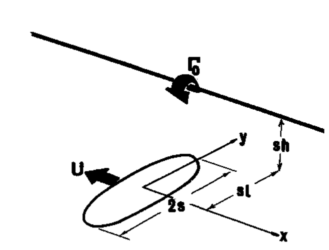

Fig. 1 Elliptic wing of aspect ratio A flying parallel to an infinite line vortex of strength Γ_o.

A LIFTING surface in uniform flight through the downwash field induced by an infinitely long vortex filament may serve to model a number of phenomena: wake penetration, wing-tail interference, formation flying, rotor aerodynamics. A solution for an important special case—flight path parallel to the vortex (Fig. 1)—was recently presented in these notes by W. P. Jones.[1] It should be remarked that for this particular case the *total* vortex induced lift and rolling moment can be obtained exactly (within the framework of linearized lifting surface theory) by use of the reciprocity relations for wings in nonuniform downwash fields.[2] The vortex induced lift corresponding to the elliptic wing case treated by Jones is, in fact, explicitly given in one of Heaslet and Spreiter's examples (Ref. 2, Eq. 59). The analogous rolling moment may also be readily deduced from their results (Ref. 2, Eq. 69).

If the spanwise distribution of the loading is required, it must be calculated directly. Jones, for example, used lifting line theory to calculate the span loading on elliptic wings in incompressible flow.[1] The rest of this Note gives these same results in an alternate simpler form more suitable for computation.

The elliptic wing of aspect ratio A and semispan s is flying parallel to a vortex of strength Γ_o displaced by lateral distance sl and vertical distance sh from midspan. It is customary to express the spanwise distribution of circulation by the Fourier series

$$\Gamma(y) = \sum_0^\infty a_n \sin(n+1)\theta \tag{1}$$

where $\theta = \cos^{-1}(y/s)$. The Fourier coefficients a_n must be such that the wake induced downwash just cancels out the vortex induced vertical velocity

$$w(y) = (\Gamma_o/2\pi s)(l+y)/[(l+y)^2+h^2] \tag{2}$$

at the wing.

Jones[1] gave a recursive method for calculating the a_n. Explicit expressions may however be obtained by means of the method of Ref. 3: using the vertical velocity distribution Eq. (2) in Eq. (8) of Ref. 3 these are

$$a_{2n} = (-1)^{n+1}\frac{8\Gamma_o}{\pi}\frac{r^{2n+1}\sin(2n+1)\phi}{A+2(2n+1)}$$

$$a_{2n+1} = (-1)^n\frac{8\Gamma_o}{\pi}\frac{r^{2n+2}\cos(2n+2)\phi}{A+2(2n+2)}$$

where

$$r^2 = h^2+l^2+G-(2)^{1/2}[l-(G-H)^{1/2}+h(\operatorname{sgn} H)(G+H)^{1/2}]$$

$$\tan\phi = \frac{l-[(G-H)/2]^{1/2}}{h-(\operatorname{sgn} H)[(G+H)/2]^{1/2}}$$

$$H = 1+h^2-l^2$$

$$G = [H^2+4h^2l^2]^{1/2}$$

Substitution into Eq. (1) gives the expression

$$\Gamma/\Gamma_o = -8/\pi\sum_0^\infty(-1)^n r^{2n+1}\left[\frac{\sin(2n+1)\phi\sin(2n+1)\theta}{A+2(2n+1)} - r\frac{\cos(2n+2)\phi\sin(2n+2)\theta}{A+(4(n+1)}\right] \tag{3}$$

for the vortex induced circulation. Values computed using Eq. (3) agree with Figs. 2 and 3 of Ref. 1.

The corresponding vortex induced increments in wing lift and rolling moment coefficients may be expressed in the form

$$\Delta C_L = C_{L_\alpha}\Gamma_o(\pi s U)^{-1}\{l-[(G-H)/2]^{1/2}\} \tag{4}$$

$$\Delta C_l = 4C_{l_p}\Gamma_o(\pi s U)^{-1}\{1+2h^2+l[2(G-H)]^{1/2} - h(\operatorname{sgn} H)[2(G+H)]^{1/2}\} \tag{5}$$

where

$$C_{L_\alpha} = 2\pi A/(A+2)$$

is the wing lift-curve slope in uniform flow, and

$$C_{l_p} = -(\pi/4)A/(A+4)$$

is the wing roll-damping derivative.

It may be verified that Eqs. (4) and (5) are identical to expressions calculated using the reciprocity relations of Ref. 2. Using appropriate values for C_{L_α} and C_{l_p} the vortex induced contributions to lift and rolling moment may be estimated from Eqs. (4) and (5) for wings of nonelliptical planform.

Finally, it should be noted that some account of the viscous core in a real vortex may be introduced in these results through the artifice of using "effective'" values for the vortex location and circulation.[4,5]

References

[1] Jones, W. P., "Vortex-Elliptic Wing Interaction," *AIAA Journal*, Vol. 10, No. 2, Feb. 1972, pp. 225–227.

[2] Heaslet, M. A. and Spreiter, J. R., "Reciprocity Relations in Aerodynamics," TN2700, 1952, NACA.

[3] Filotas, L. T., "Solution of the Lifting Line Equation for Twisted Elliptic Wings," *Journal of Aircraft*, Vol. 8, No. 10, Oct. 1971, pp. 835–836.

[4] Johnson, W., "Application of a Lifting-Surface Theory to the Calculation of Helicopter Airloads," Preprint No. 510, presented at the 27th Annual National V/STOL Forum of the American Helicopter Society, Washington, D.C., May 1971.

[5] Widnall, S., "Helicopter Noise due to Blade-Vortex Interaction," *Journal of the Acoustical Society of America*, Vol. 50, No. 1, Pt. 2, 1971.

Received February 17, 1972.
Index category: Airplane and Component Aerodynamics.
* Assistant Professor, Department of Aerospace Engineering.

Computation of Transonic Flow about Finite Lifting Wings

P. A. Newman* and E. B. Klunker*
NASA Langley Research Center, Hampton, Va.

RELAXATION methods have proven to be an accurate and effective means for computing the transonic flowfield around two-dimensional airfoils when the freestream is subsonic. Murman and Cole[1] have developed such a technique for solving

Received February 25, 1972.
Index category: Subsonic and Transonic Flow.
* Aerospace Technologist, Loads Division.